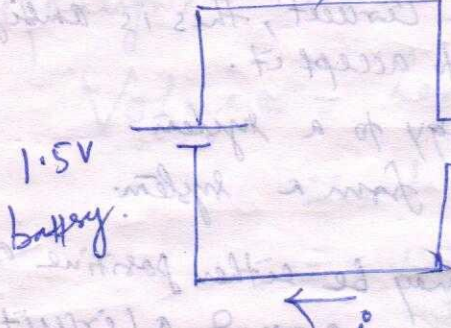


# KIRCHHOFF'S CURRENT LAW: →

$i =$  current flowing in closed circuit.



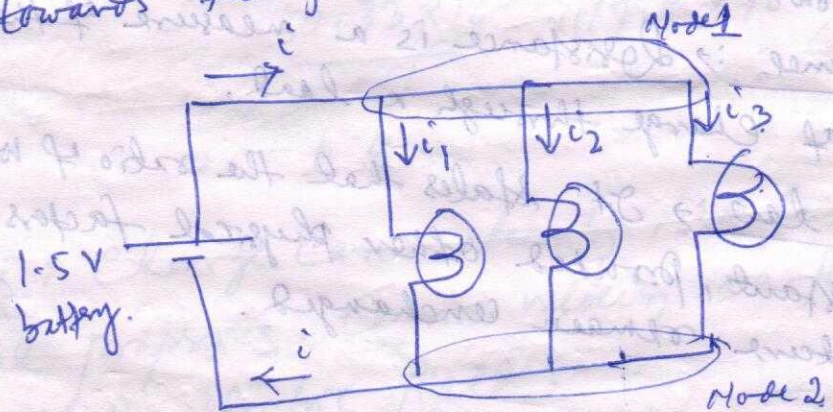
In the circuit, the current  $i$  flowing from the battery through the light bulb is equal to the current flowing from the light bulb to the battery. In other words, no current is "lost" around the closed circuit.

This principle was observed by the German Scientist (G.R. Kirchhoff and is known as Kirchhoff's Current Law (KCL).

Kirchhoff's Current Law states that because charge cannot be created but must be conserved, at any instant the algebraic sum of the currents at a node or junction in a network is zero. Formally,

$$\sum_{n=1}^N i_n = 0 \quad \text{Kirchhoff's current law.}$$

Different signs are allocated to currents - held flow towards the junction and those away from it.



In the above circuit, if we define the currents entering a node as positive and the currents exiting the node as being negative, then the resulting expression for node 1 by applying KCL is as follows:

$$i_1 - i_2 - i_3 = 0$$

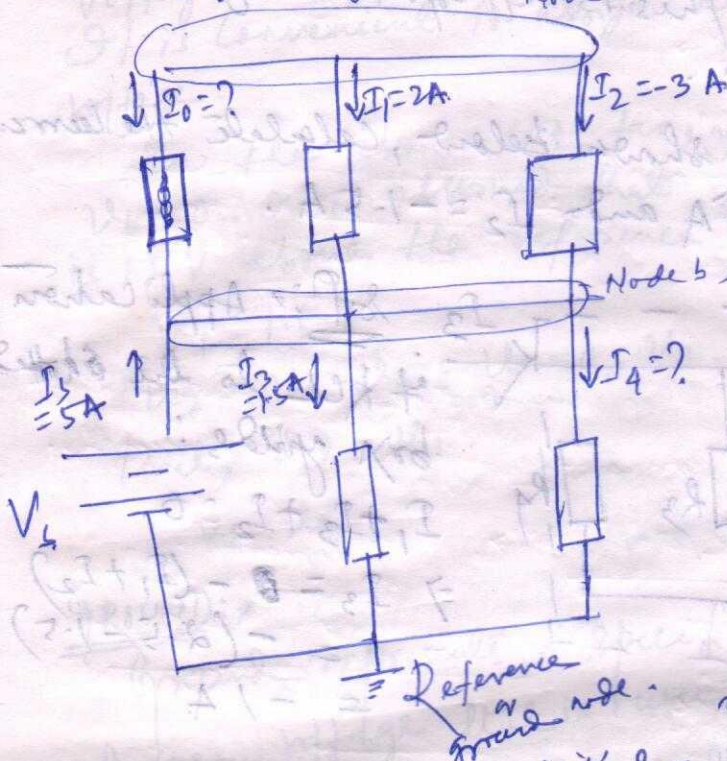
If we had assumed that currents entering the node were negative, the result would not have changed.



Ex: 2.3 APPLICATION OF KCL: →

Q-22

Problem: → Determine the unknown currents in the circuit of the figure given below.



Sol: → Applying KCL at node 'a'

$$-I_0 - I_1 - I_2 = 0$$

$$\Rightarrow I_0 = -(I_1 + I_2)$$

$$= -(2 - 3)$$

$$= 1 \text{ A.}$$

Applying KCL at node 'b'

$$I_3 - I_4 = 0$$

$$\Rightarrow I_4 = I_3 - I_5$$

$$= 5 - 1.5$$

$$= 3.5 \text{ A.}$$

Alternatively applying KCL at reference or ground node.

$$I_3 + I_4 - I_5 = 0$$

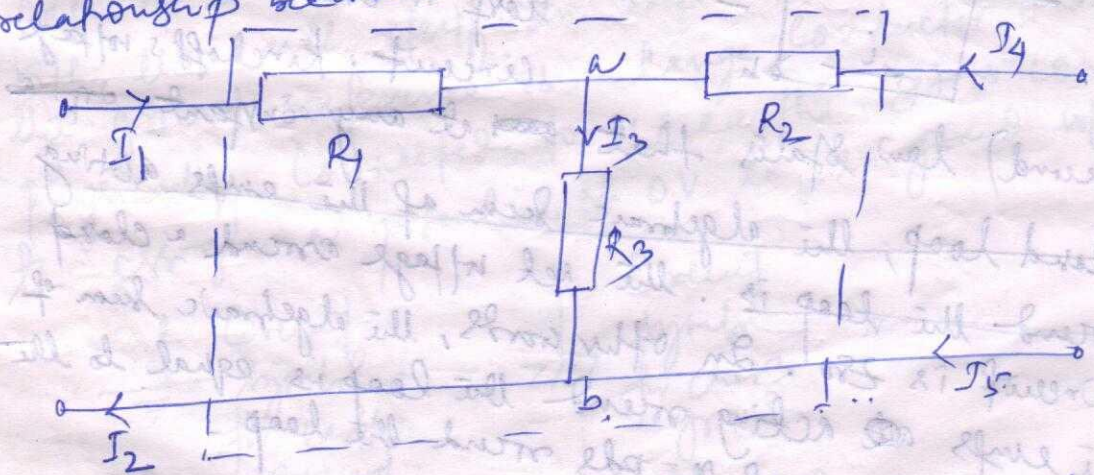
$$\Rightarrow I_4 = I_5 - I_3 = 5 - 1.5$$

$$= 3.5 \text{ A}$$

Three nodes are: node a, node b & Reference or ground node.

We develop a method called Node Analysis to solve this kind of problem in the foregoing analysis with reference to the network shown below, determine the relationship between the currents  $I_1, I_2, I_4$  and  $I_5$ .

Ex: 2.4  
H-13



Sol: → for node 'a':

$$I_1 + I_4 - I_3 = 0$$

$$\Rightarrow I_3 = I_1 + I_4 \quad \text{--- (1)}$$

for node 'b':

$$I_3 + I_5 - I_2 = 0$$

$$\Rightarrow I_2 = I_3 + I_5 \quad \text{--- (2)}$$

From eqn (1) & (2), we have:

$$I_1 + I_4 = I_2 - I_5$$

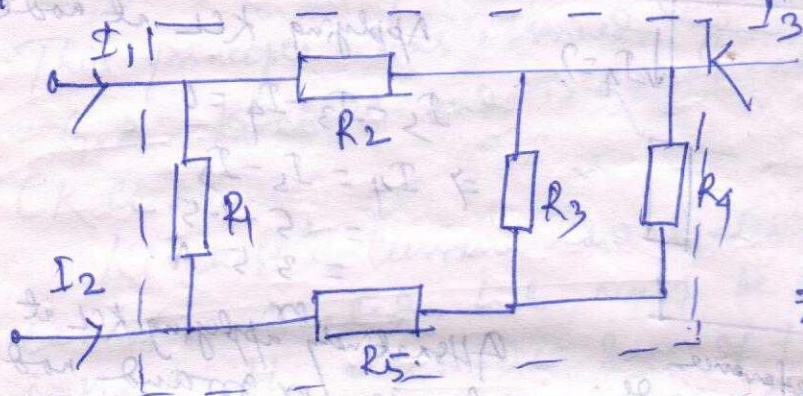
$$\Rightarrow I_1 - I_2 + I_4 + I_5 = 0$$

from the result of this example; it may be noted that KCL need not only apply to a junction or a node but also to a section of



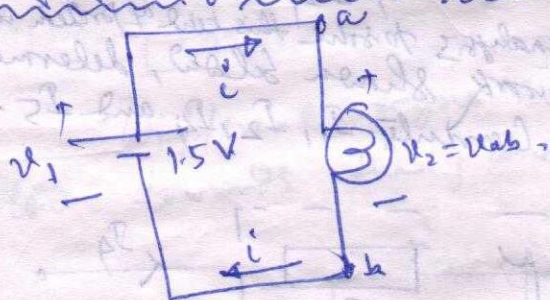
It follows that knowledge of the performance of a quite a complicated network may not be required if only the input and output quantities are to be investigated. This is illustrated by the following problem.

Ex: 3.10  
 H-44 For the network shown below, calculate the current  $I_3$  if  $I_1 = 2.5 \text{ A}$  and  $I_2 = -1.5 \text{ A}$ .



App: Application of KCL to the shaded box yields:  
 $I_1 + I_3 + I_2 = 0$   
 $\Rightarrow I_3 = -(I_1 + I_2)$   
 $= -(2.5 - 1.5)$   
 $= -1 \text{ A}$

### KIRCHOFF'S VOLTAGE LAW $\rightarrow$



The principle underlying KVL is that no energy is lost or created in an electric circuit. Kirchhoff's voltage

(second) law states that at any instant in a closed loop, the algebraic sum of the emfs acting around the loop is zero. In other words, the algebraic sum of the emfs acting around the loop is equal to the algebraic sum of the pd's around the loop.

Formally:

$$\sum_{n=1}^N \epsilon_n = 0 \quad \text{Kirchhoff's voltage law}$$

$\epsilon_n$  are the individual voltages



REFERENCE VOLTAGE:  $\rightarrow$  In a circuit, any node may be chosen as the reference node, such that all node voltages may be referenced to this reference voltage. It is convenient to assign a value of 250 to reference voltage.

In the above ckt, taking 'b' as the reference node,  $V_b = 0$ . It is observed that the battery's positive terminal is 1.5V above the reference voltage. So we can write

$$V_1 = 1.5 \text{ V.}$$

$$V_2 = V_a - V_b = V_a - 0 = V_a.$$

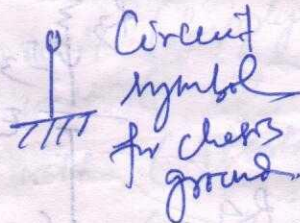
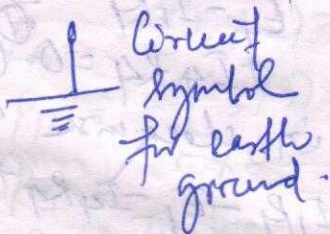
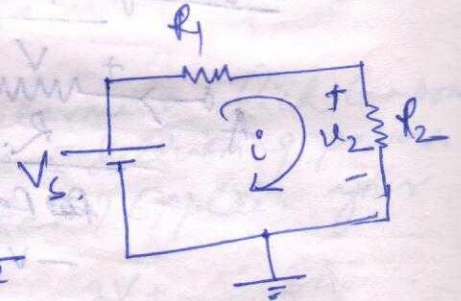
$$\text{But, } V_a = V_1.$$

$$\therefore V_1 = V_2.$$

GROUND:  $\rightarrow$

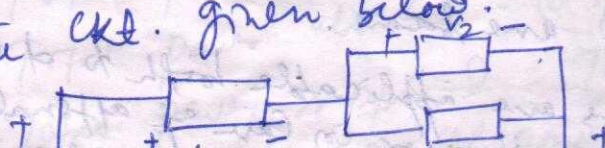
Ground represents a specific reference voltage that is usually a clearly identified point in a circuit.

For example, the ground reference voltage can be identified with the case or enclosure of an instrument, or with the earth itself. In residential electric circuits, the ground reference is a large conductor that is physically connected to the earth. It is convenient to assign a potential of 0V to the ground voltage reference.



Ex-2.6  
R-37  
Application of KVL

Determine the unknown voltage  $V_2$  by applying KVL to the ckt. given below.



Applying KVL around the simple loop, we have:

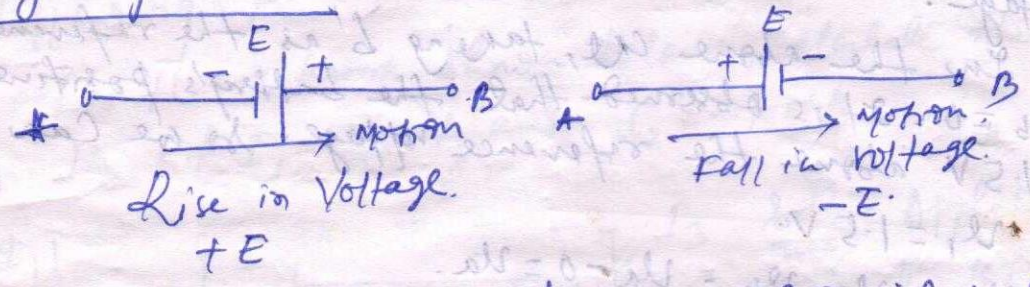
$$V_{s2} - V_1 - V_2 - V_3 = 0$$



~~Electric~~ we solve this kind of problem by developing a systematic procedure called mesh analysis.

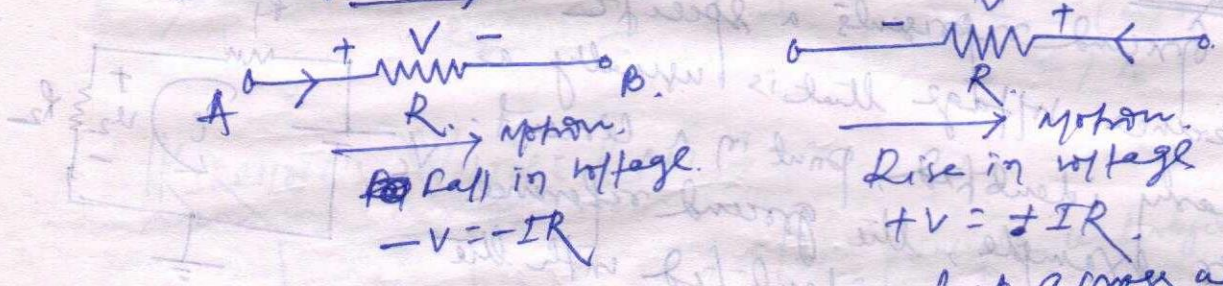
## DETERMINATION OF VOLTAGE SIGN :->

### Sign of EMF :->

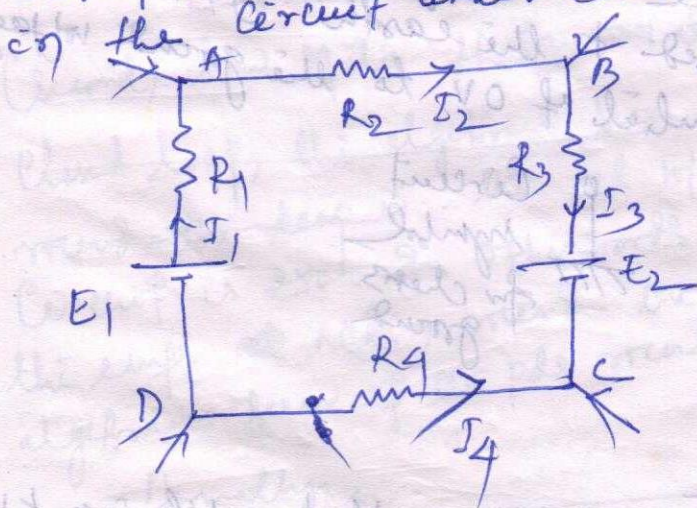


Notes :-> The sign of the battery emf is independent of the direction of current through that branch.

### Sign of Voltage drop :->



Notes :-> The sign of the voltage drop across a resistor depends on the direction of current through the resistor but is independent of any other source emf in the circuit under consideration.



Ex :-> for the closed path ABCD applying KVL, we have

$$E_1 - I_1 R_1 - I_2 R_2 - I_3 R_3 + I_4 R_4 = 0 \quad \text{① (Travelling in clockwise direction)}$$

$$I_1 R_1 - E_1 - I_4 R_4 + E_2 + I_3 R_3 = 0 \quad \text{② (Travelling in anticlockwise direction)}$$

→  $E_1 - I_1 R_1 - I_2 R_2 - I_3 R_3 - E_2 + I_4 R_4 = 0$  (Multiplying ② by -1)

∴ Eqn ① & eqn ② are same.

∴ Kirchoff's laws are applicable both to d.c. and a.c. circuits.



# ELECTRIC POWER AND SIGN CONVENTION:

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{\text{work}}{\text{charge}} \times \frac{\text{charge}}{\text{time}} = \text{Voltage} \times \text{Current}$$

$$\left( \frac{\text{Joules}}{\text{Coulomb}} \times \frac{\text{Coulomb}}{\text{sec}} \right) = \text{joules/sec} = \text{watt}$$

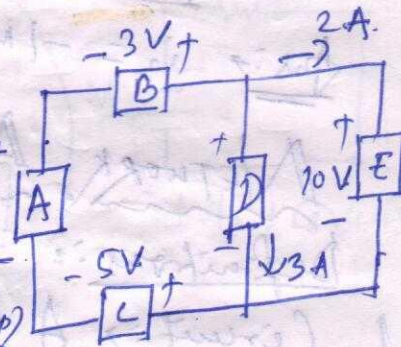
Just like voltage, power is a signed quantity & it is necessary to make a distinction between positive and negative power.

## PASSIVE SIGN CONVENTION:

The power dissipated by a load is a positive quantity. If current flows from a higher to a lower voltage (plus to minus), the power is dissipated and will be positive quantity and vice-versa.

Ex: 2.9  
R-1)

For the circuit shown below determine which components are absorbing power and which are delivering power. Is conservation of power satisfied? Explain your answer.



Sol: Applying KCL, the current through element B is 5A to the right. By KVL,

$$V_A + 3 - 10 - 5 = 0 \Rightarrow V_D = V_E = 10V \text{ (positive at the top)}$$

$$\Rightarrow V_A = 12V \text{ (positive at the top)}$$

$$P_A = -(5 \times 12) = -60 \text{ watt (supplying power)}$$

$$P_B = -(5 \times 3) = -15 \text{ watt (supplying power)}$$

$$P_C = 5 \times 5 = 25 \text{ watt (absorbing power)}$$

$$P_D = (10 \times 3) = 30 \text{ watt (absorbing power)}$$

$$P_E = (10 \times 2) = 20 \text{ watt (absorbing power)}$$

$$\text{Total power supplied} = 60 + 15 = 75 \text{ watts}$$

$$\text{Total power absorbed} = (25 + 30 + 20) = 75 \text{ watts}$$

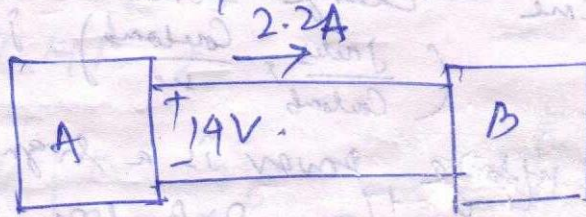
Total power supplied = Total power absorbed,

conservation of power is satisfied.



Pb: 7  
R-4

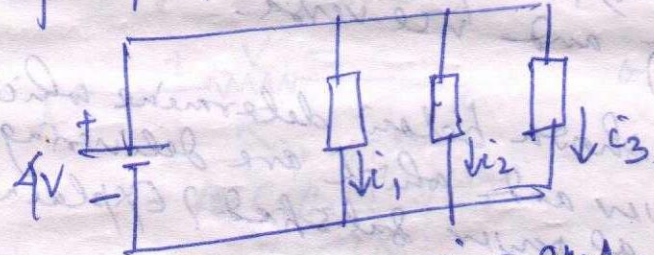
Determine which circuit element in the following question is supplying power and which is dissipating. Also determine the power dissipated and supplied.



Ans:  $\rightarrow$  A supplies  $30.8 \text{ W}$  & B dissipates  $30.8 \text{ W}$ .

Pb: 7  
R-42

If the battery in the following diagram supplies a total of  $10 \text{ mW}$  to the three elements shown and  $i_1 = 2 \text{ mA}$  and  $i_2 = 1.5 \text{ mA}$ , what is the current  $i_3$ ?  
if  $i_1 = 1 \text{ mA}$  and  $i_3 = 1.5 \text{ mA}$ , what is  $i_2$ ?



Ans:  $\rightarrow i_3 = -1 \text{ mA}, i_2 = 0 \text{ mA}$ .

NETWORK ANALYSIS

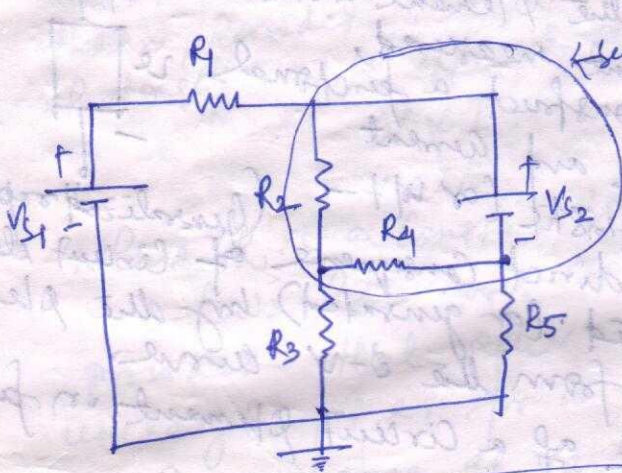
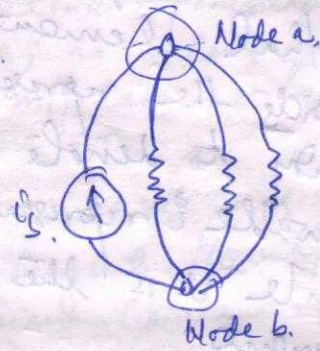
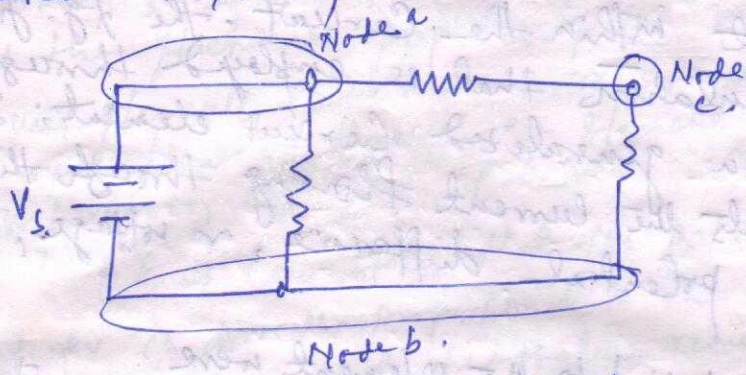
ANALYSIS  $\rightarrow$  The analysis of an electric network consists of determining each of the unknown branch currents and node voltages.

Definitions:

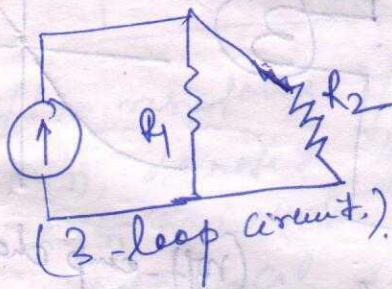
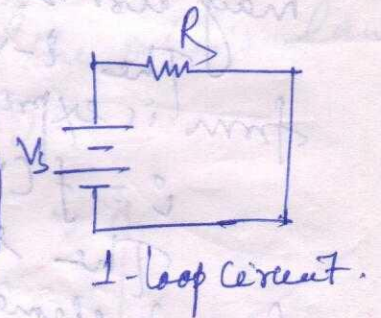
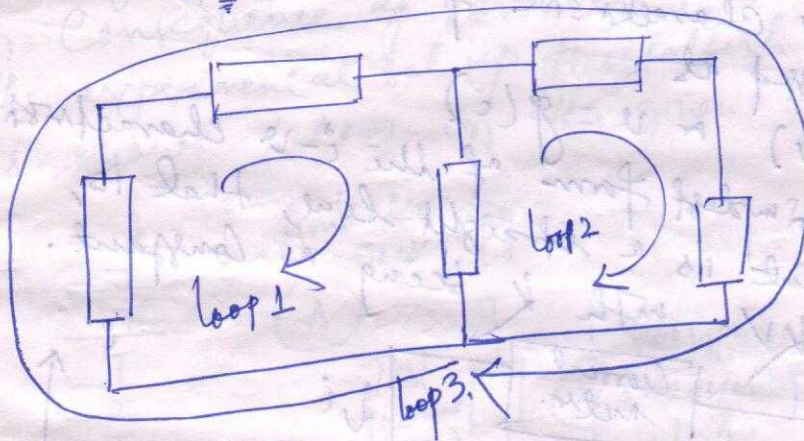
- Circuit  $\rightarrow$  A circuit is a closed conducting path through which an electric current either flows or is intended to flow. Circuit consists of active and passive elements in it.
  - Electrical network  $\rightarrow$  An electrical network is a collection of elements through which current flows.
- Definitions of some important elements of a network are as follows:
- Node  $\rightarrow$  It is a point in a circuit where two or more circuit elements are connected together.
  - Branch  $\rightarrow$  It is that part of a network which lies between two nodes.
  - Loop  $\rightarrow$  It is a closed path in a network. A loop is any closed connection of branches. A loop is any closed path which does not contain other loops.



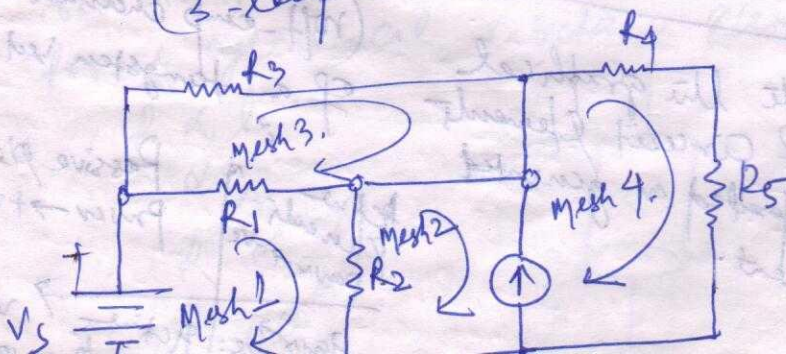
Supernode  $\rightarrow$  A supernode is obtained by defining a region that encloses more than one node. Supernodes can be treated in exactly the same way as nodes.



(Definitions of node & supernode)



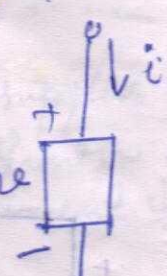
(Definition of a loop)





# CIRCUIT ELEMENTS AND THEIR $i-v$ CHARACTERISTICS: $\rightarrow$

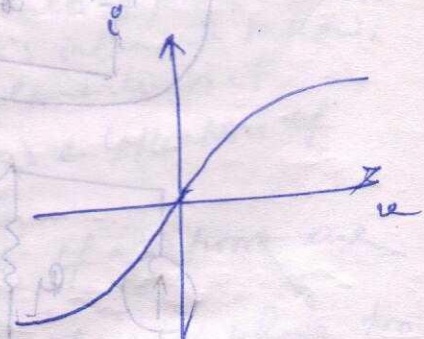
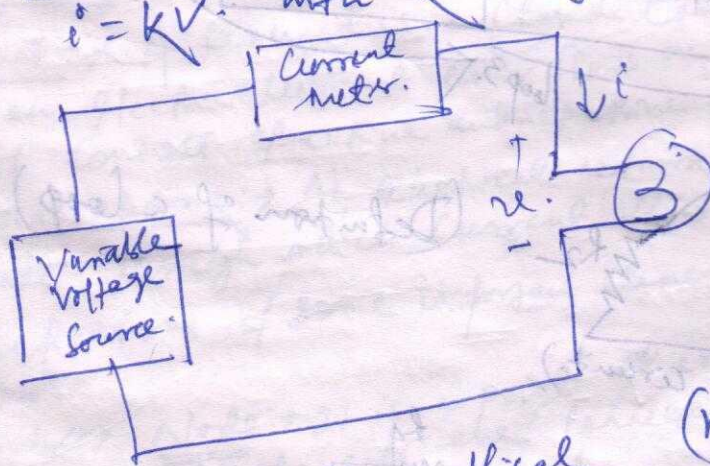
The relationship between current & voltage at the terminals of a circuit element defines the behaviour of that element within the circuit. The fig. given below depicts the representation that is employed throughout our study to denote a generalized circuit element: The variable  $i$  represents the current flowing through the element, while  $v$  is the potential difference, or voltage, across the element.



If the voltage applied to the element were varied and the resulting current measured, it would be possible to construct a functional relationship between voltage and current known as the  $i-v$  characteristic (or  $v-i$  dependent characteristic). A direct consequence is that the power dissipated (or generated) by the element may also be determined from the  $i-v$  curve.

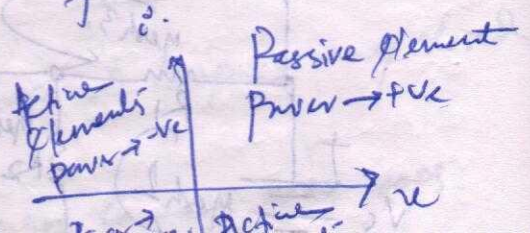
The  $i-v$  characteristic of a circuit element or functional form is expressed as:  
 $i = f(v)$  or  $v = g(i)$

The simplest form of the  $i-v$  characteristic for a circuit element is a straight line, that is,  
 $i = kV$ , with  $k$  being a constant.



( $v-i$  characteristic of a tungsten bulb)

We can also relate the graphical  $i-v$  representation of circuit elements to the power dissipated or generated by a circuit element:





# RESISTANCE AND OHM'S LAW →

Resistance is a measure of the opposition to the flow of current through a conductor, the magnitude of which depends on the electrical properties of the material. Practically all circuit elements exhibit some resistance; as a consequence, current flowing through an element will cause energy to be dissipated in the form of heat.

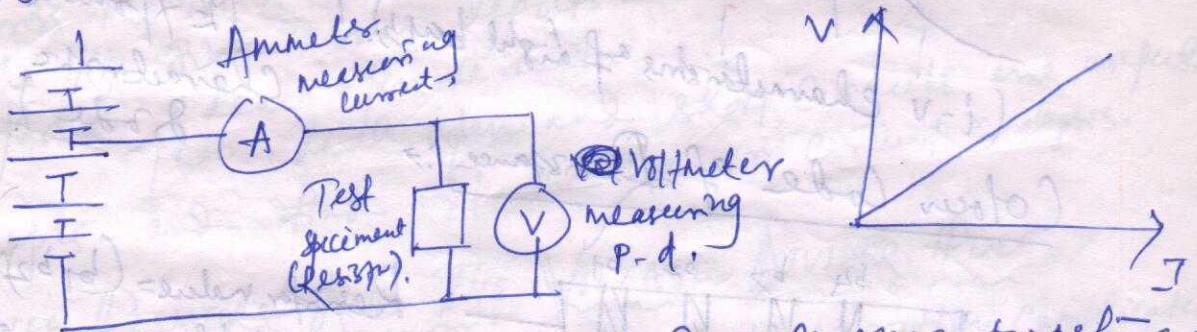
Ohm's law states that the p.d. across the ends of a conductor is directly proportional to the current flowing between them, provided that temperature remained constant. The relationship may be expressed as:

$$V \propto I$$

$$\Rightarrow V = IR$$

where  $R$  is a constant termed the resistance of the conductor.

Ohm's law notes the constancy of p.d. to current provided that other physical factors remain unchanged, i.e. for a given p.d. the current will vary in consequence of variation of physical factors. The experimental set up to establish the ohm's law is as shown below in the given fig.

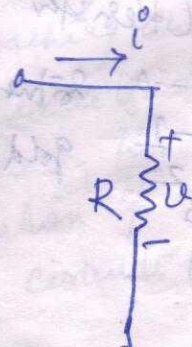
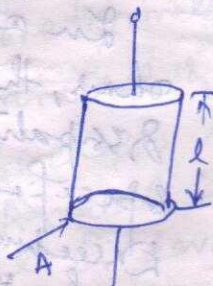


The resistance of a material depends on a property called resistivity, denoted by the symbol  $\rho$ , the inverse of resistivity is called conductivity  $\sigma$  is denoted by symbol  $\sigma$ . For a cylindrical resistance element

$$R \propto l$$

$$R \propto \frac{1}{A}$$

$$R \propto \frac{l}{A}$$





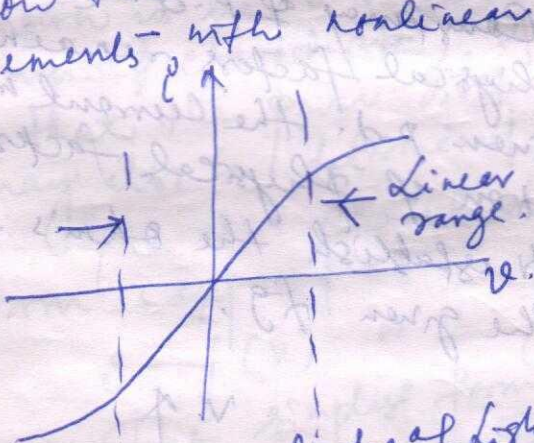
Inverse of Resistance is called conductance.

$$G = \frac{1}{R} \text{ Siemens (S)}$$

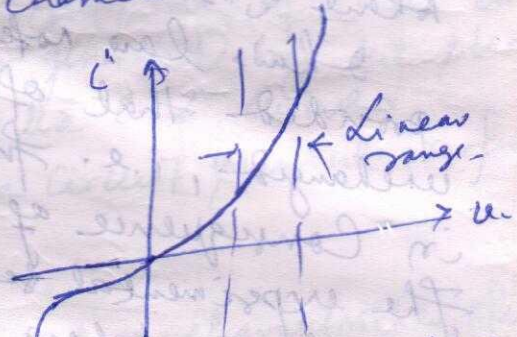
Thus, Ohm's law can be restated in terms of Conductance as:

$$I = GV$$

Typically, the linear relationship between voltage & current in electrical conductors does not apply at very high voltages and currents. Further, not all electrically conducting materials exhibit linear behaviour even for small voltages & currents. It is usually true, however, that for some range of voltages and currents, most elements display a linear  $i-v$  characteristic. The fig. given below illustrates how the linear resistance concept may apply to elements with nonlinear  $i-v$  characteristics.

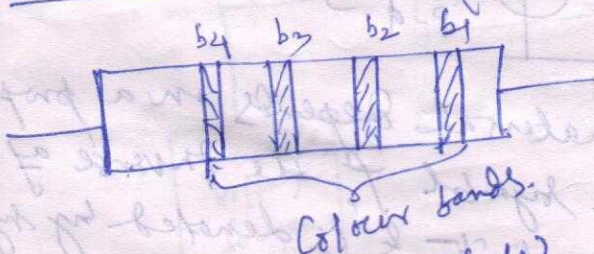


( $i-v$  characteristics of light bulb)



(Exponential  $i-v$  characteristic of semiconductor diode)

### Colour Code of Resistance $\Rightarrow$



Colour bands

(Resistor Colour Code)

$$\text{Resistor value} = (b_1 b_2) \times 10^{b_3}$$

$b_4 = \% \text{ tolerance in actual value.}$

Ex:  $b_1 = \text{brown}, b_2 = \text{black}, b_3 = \text{black}$

$$R = (10) \times 10^0 = 10 \Omega$$

- black - 0
- brown - 1
- red - 2
- orange - 3
- yellow - 4
- green - 5
- blue - 6
- gold - 5%
- silver - 10%

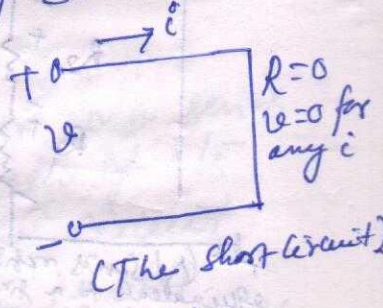
In addition to the resistance in ohms, the maximum allowable power dissipation (or power rating) is typically specified for commercial resistors. Exceeding this power rating leads to overheating and can cause the resistor to fail. This power dissipated



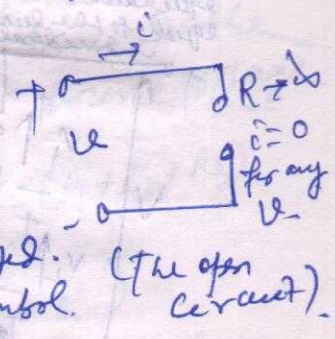
# OPEN & SHORT CIRCUITS: →

Two convenient idealizations of the resistance element are provided by the limiting cases of Ohm's law as the resistance of a circuit element approaches zero or infinity, i.e. short circuit & open circuit respectively.

Formally, a short circuit is defined as a circuit element across which the voltage is zero, regardless of the current flowing through it. The fig. given below depicts the circuit symbol for an ideal short circuit. It follows that if two points are connected by a conductor of zero resistance they are said to be short-circuited.



In the second instance, a circuit element whose resistance approaches infinity is called an open circuit. An open circuit is defined as a circuit element through which the current is zero, regardless of the voltage applied across it. Effectively there is no connection between the two points in an open circuit. For example, if the wire of a circuit is broken, then there is no connection across the break and the circuit is open-circuited. The fig. given above depicts the circuit symbol for an ideal open-circuit.



The ideal open and short circuits are useful concepts and find extensive use in circuit analysis.

## SERIES RESISTORS AND THE VOLTAGE DIVIDER RULE: →

Although electric circuits can take rather complicated forms, but even the most complicated circuits can be reduced to combination of circuit elements in parallel and in series. So it is important to get acquainted with parallel & series circuits before formally approaching the topic of network analysis. Parallel & series circuits have a direct relationship with Kirchhoff's laws. These two circuits are based on series & parallel combination of resistors: the voltage divider and current divider. These circuits form the



Definitions →

Two or more circuit elements are said to be in series if the current from one element exclusively flows into the next element. From KCL, it then follows that all series elements have the same current.

By applying KVL, it can be verified that the sum of the voltages across the resistors equals the voltage externally provided by the battery.

$$V = V_1 + V_2 + V_3$$

Since, in general,  $V = IR$ , then  $V_1 = IR_1$ ,  $V_2 = IR_2$  &  $V_3 = IR_3$ , the current  $I$  being the same in each resistor.

$$V = IR_1 + IR_2 + IR_3$$

For the complete circuit, the effective resistance of the load  $R$  represents the ratio of the supply voltage to the circuit current, hence

$$V = IR$$

$$\text{but } V = IR_1 + IR_2 + IR_3$$

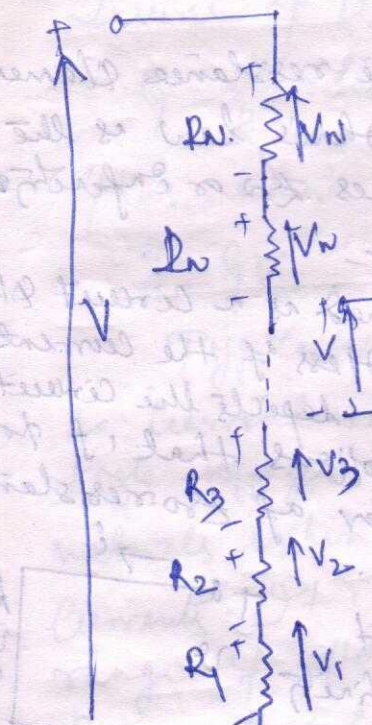
$$\Rightarrow IR = IR_1 + IR_2 + IR_3$$

$$\Rightarrow R = R_1 + R_2 + R_3$$

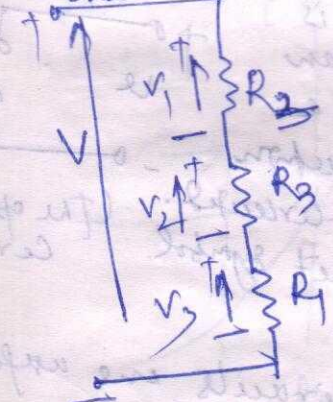
The three resistors could thus be replaced by a single resistor of value  $R$  without changing the amount of current required of the battery. From this result we may extrapolate to the more general relationship defining the equivalent resistance of  $N$  series resistors -

$$R = \sum_{n=1}^N R_n \text{ Equivalent series resistance}$$

A concept very closely associated with series resistors is that of the voltage divider. This terminology originates from the observation that the source voltage in series circuit divides among the three resistors according to KVL. For the three resistors connected in series, the resistance of the circuit is



(N) Series resistors are equivalent to a single resistor equal to the sum of the individual resistances.





and therefore, the current in the circuit is

$$I = \frac{V}{R_1 + R_2 + R_3}$$

The voltage drop across  $R_1$ ,  $R_2$  and  $R_3$  is respectively given by,

$$V_1 = IR_1 = \frac{V}{R_1 + R_2 + R_3} R_1 = \frac{R_1}{R} V$$

$$V_2 = IR_2 = \frac{V}{R_1 + R_2 + R_3} R_2 = \frac{R_2}{R} V$$

$$\& V_3 = IR_3 = \frac{V}{R_1 + R_2 + R_3} R_3 = \frac{R_3}{R} V$$

The voltage across each resistor in a series circuit is directly proportional to the ratio of its resistance to the total resistance of the circuit.

It can be verified that KVL is still satisfied, by adding the voltage drops around the circuit & equating their sum to the source voltage.

$$V_1 + V_2 + V_3 = \frac{R_1}{R} V + \frac{R_2}{R} V + \frac{R_3}{R} V = \frac{V}{R} (R_1 + R_2 + R_3)$$

$$= \frac{V}{R} \times R = V \quad (\because R_1 + R_2 + R_3 = R)$$

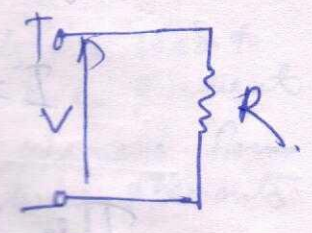
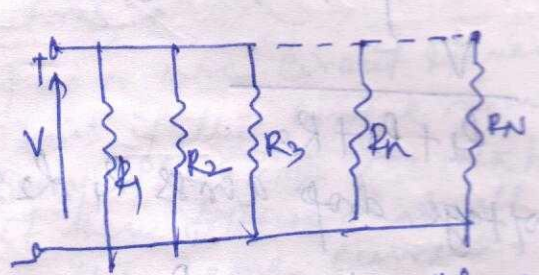
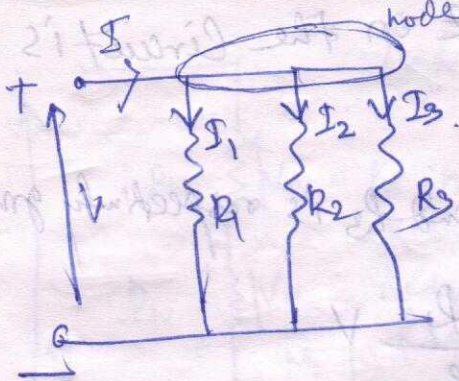
By virtue of the voltage divider rule, we can always determine the proportion in which voltage drops are distributed around a circuit. The general form of the voltage divider rule for a circuit with  $N$  series resistors and a voltage source is

$$V_n = \frac{R_n}{R_1 + R_2 + \dots + R_n + \dots + R_N} V_s \quad \text{Voltage divider}$$

~~Parallel~~

**PARALLEL RESISTORS AND THE CURRENT DIVIDER RULE:**  $\rightarrow$   
 Definition:  $\rightarrow$  Two or more circuit elements are said to be in parallel if all elements share the same voltage across them. All elements will have the same voltage across them.





(N resistors in parallel are equivalent to a single equivalent resistor with resistance equal to inverse of the sum of the inverse of the individual resistances.)

Applying KCL at top node,  $I = I_1 + I_2 + I_3$ .

Since in general,  $I = \frac{V}{R}$ , then,  $I_1 = \frac{V}{R_1}$ ,  $I_2 = \frac{V}{R_2}$  and  $I_3 = \frac{V}{R_3}$ .  
The voltage across each branch being the same.

$$\therefore I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

For the complete network the effective resistance of the load represents the ratio of the supply voltage to the supply current, whence  $I = \frac{V}{R}$ .

Hence,  $I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

$$\Rightarrow \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Generalizing the above

result, it can be said that ~~N~~ N resistors connected in parallel across a source, act as a single equivalent resistance R given by the expression.

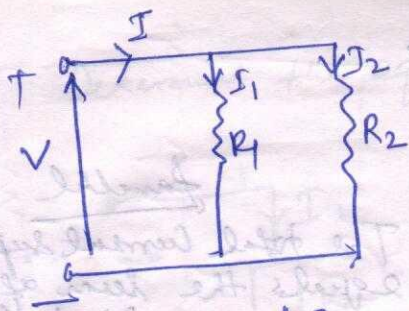
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} + \dots + \frac{1}{R_N}$$

$$\Rightarrow R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} + \dots + \frac{1}{R_N}}$$

Equivalent Parallel Resistance

In further discussion, very often we refer to the parallel combination of two or more resistors with the symbol  $\parallel$  signifies





Circuit with two resistors in parallel.

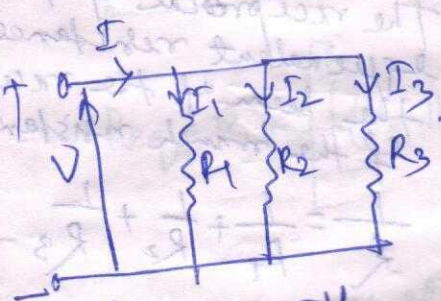
For two parallel connected resistor, the effective resistance  $R$  is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R = \frac{R_1 R_2}{R_1 + R_2}$$

But,  $V = IR = I R_1$

$$\Rightarrow I R_1 = I \cdot \frac{R_1 R_2}{R_1 + R_2}$$

$$\Rightarrow I_1 = I \cdot \frac{R_2}{R_1 + R_2}$$



(Circuit with three resistors in parallel)

For three parallel connected resistor, we have:  $I_1 = \frac{V}{R_1}$ ,  $I_2 = \frac{V}{R_2}$  &  $I_3 = \frac{V}{R_3}$

and since,  $V = R I$ , the currents in each resistor is given by:

$$I_1 = \frac{R I}{R_1} = \frac{1/R_1}{1/R_1 + 1/R_2 + 1/R_3} I$$

$$I_2 = \frac{R I}{R_2} = \frac{1/R_2}{1/R_1 + 1/R_2 + 1/R_3} I$$

$$I_3 = \frac{R I}{R_3} = \frac{1/R_3}{1/R_1 + 1/R_2 + 1/R_3} I$$

From the above result, it is concluded that, the current in a parallel circuit divides in inverse proportion to the resistances of the individual parallel elements. The general expression for the current divider for a circuit with  $N$  parallel resistors is given by:

$$i_n = \frac{1/R_n}{1/R_1 + 1/R_2 + \dots + 1/R_n + \dots + 1/R_N} I \quad \text{Current divider.}$$



# SERIES Vs PARALLEL CIRCUITS:

## Series

Current  $\rightarrow$  Current is same in all parts of the circuit,  
i.e.  $I = I_1 = I_2 = I_3$

Voltage  $\rightarrow$  The total voltage equals the sum of the voltages across each element.  
 $V = V_1 + V_2 + V_3$

Resistance  $\rightarrow$  The total resistance equals the sum of the individual resistances,  $R = R_1 + R_2 + R_3$ .

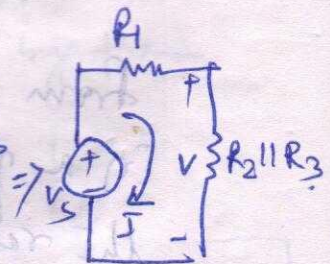
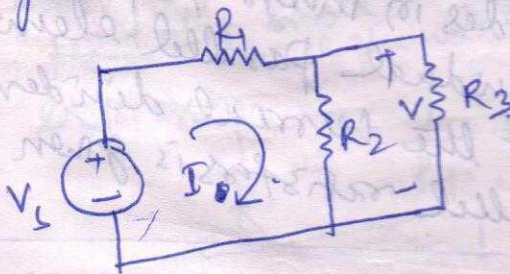
## Notes

- 1) In a series circuit, the total resistance is always greater than the greatest resistance in the circuit.
- 2) In a parallel circuit, the total resistance is always less than the smallest resistance in the circuit.

# SERIES-PARALLEL CIRCUIT:

Determine the voltage  $V$  in the circuit of the fig.

Pb:  
 $R = 5\Omega$  given below.

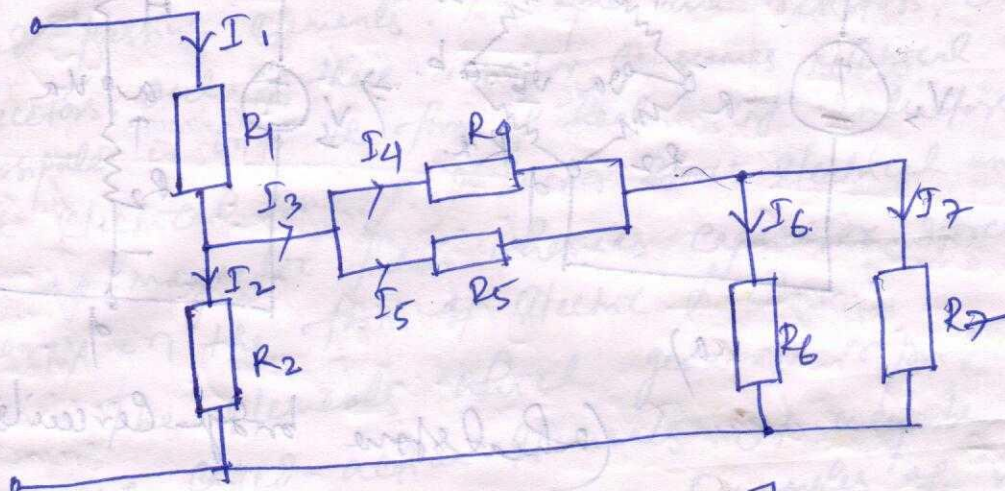


$$V = \frac{R_2 || R_3}{R_1 + (R_2 || R_3)} V_s \quad \& \quad I = \frac{V_s}{R_1 + (R_2 || R_3)}$$



Pb:  $\rightarrow$   
H-45

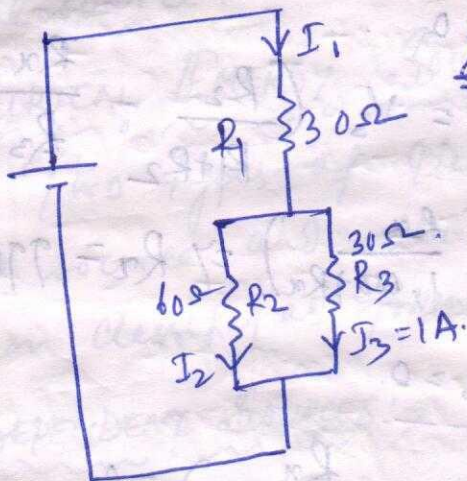
Determine the equivalent resistance of the CKT. given below.



$$R_{eq} = R_1 + \left[ (R_4 \parallel R_5) + (R_6 \parallel R_7) \parallel R_2 \right]$$

Pb:  $\rightarrow$  Ex  
H-46

For the circuit shown below, determine  $I_1$  &  $I_2$ .



Sol:  $\rightarrow$   $I_3 = I_1 \cdot \frac{R_2}{R_2 + R_3}$

$$\Rightarrow 1 = I_1 \cdot \frac{60}{60 + 30}$$

$$\Rightarrow I_1 = \frac{90}{60} = 1.5 \text{ A}$$

$$\therefore I_1 = I_2 + I_3 \Rightarrow I_2 = I_1 - I_3 = 1.5 - 1 = 0.5 \text{ A}$$

Pb:  $\rightarrow$  Ex  
R-55

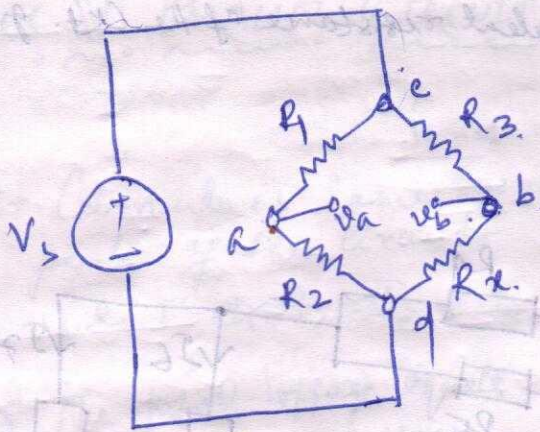
The Wheatstone bridge as shown in the fig. given below is a resistive circuit that is frequently encountered in a variety of measurement circuits.

1. For the circuit determine the voltage  $V_{ab} = V_{ad} - V_{bd}$  in terms of the four resistances and the source voltage  $V_s$ . It should be noted that since the reference point d is the same for both voltages, we can also write  $V_{ab} = V_a - V_b$ .

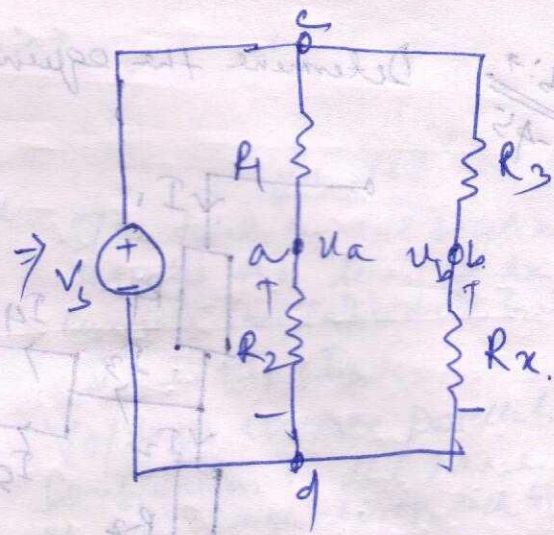
2. If  $R_1 = R_2 = R_3 = 1 \text{ k}\Omega$ ,  $V_s = 12 \text{ V}$ , and  $V_{ab} = 12 \text{ mV}$ , determine the value of  $R_x$ .



Sol.



(a)



(Wheatstone bridge circuits)

$$1. V_{ad} = V_s \cdot \frac{R_2}{R_1 + R_2} \quad \text{and} \quad V_{bd} = V_s \cdot \frac{R_x}{R_3 + R_x}$$

Applying KVL to loop abda:

$$-V_{ad} + V_{bd} + V_{ab} = 0$$

$$\Rightarrow V_{ab} = V_{ad} - V_{bd} = V_s \left( \frac{R_2}{R_1 + R_2} - \frac{R_x}{R_3 + R_x} \right)$$

$$2. 0.012 = 12 \left( \frac{1000}{2000} - \frac{R_x}{1000 + R_x} \right) \Rightarrow R_x = 996 \Omega$$

Notes:  $\Delta$  Conditions for  $V_{ab} = 0$

$$V_{ab} = 0 \Rightarrow V_s \left( \frac{R_2}{R_1 + R_2} - \frac{R_x}{R_3 + R_x} \right) = 0$$

$$\Rightarrow \frac{R_2}{R_1 + R_2} = \frac{R_x}{R_3 + R_x} \Rightarrow R_2 R_3 + R_2 R_x = R_1 R_x + R_2 R_x$$

$$\Rightarrow \boxed{R_2 R_3 = R_1 R_x}$$

2. The application of KVL need not be restricted to actual circuits. Instead, part of a circuit may be imagined and KVL can be applied as solved in the above problem.



## PASSIVE AND ACTIVE ELEMENTS:

All elements which consume or store electrical energy are called passive elements. Examples are resistors, inductors and capacitors. Out of these resistor consumes electrical energy & dissipates it into the form of heat energy. Inductors and capacitors store electrical energy. Inductor stores electrical energy in the form of magnetic field whereas capacitor stores electrical energy in the form of electric field.

The elements which generate or produce electrical energy are called active elements. It may be either a voltage or a current source. Examples of voltage sources are batteries, generators etc. whereas most of the semiconductor devices like transistors etc. are treated as current source. A battery charger is a device that can operate as a current source.

## ELECTRICAL ENERGY SOURCES:

Two types of electrical energy sources are: voltage source and the current source. They can be further classified as follows:

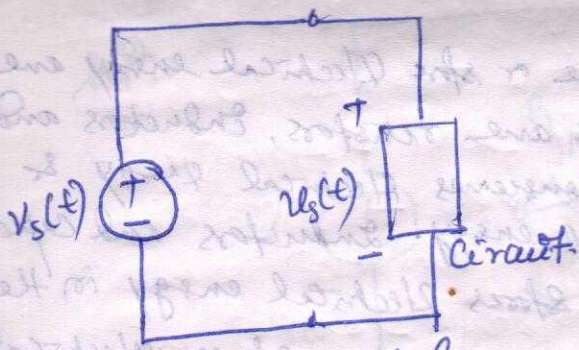
### INDEPENDENT SOURCES:

#### A. The voltage source:

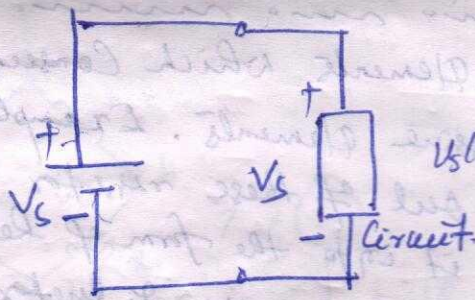
i) Ideal Voltage Source: → An ideal voltage source provides a prescribed constant voltage across its terminals irrespective of the current flowing through it. The amount of current supplied by the source is determined by the circuit connected to it.

The fig. given below depicts the various symbols for ideal voltage sources. It should be noted that by convention the direction of positive current flow out of a voltage source is out of the positive terminal.

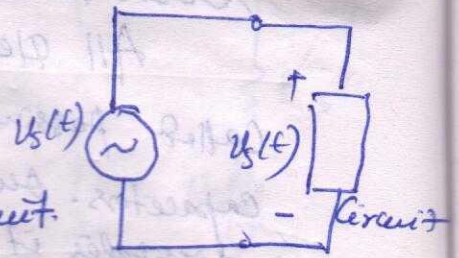




General symbol for ideal voltage source,  $V_s(t)$  may be constant (DC source) or time varying.



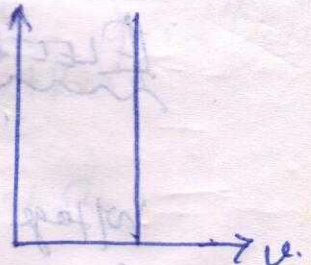
A special case: DC voltage source (Ideal battery).



A special case: Time varying source. One example is the alternating voltage source,  $V_s(t) = V_m \cos \omega t$ .

### (Ideal Voltage Sources).

An ideal voltage source generates a prescribed voltage independent of the current drawn from the load; thus, its  $i-v$  characteristic is a straight vertical line with a voltage axis intercept corresponding to the source voltage as shown below. The internal resistance of an ideal voltage source is always  $0 \Omega$ .



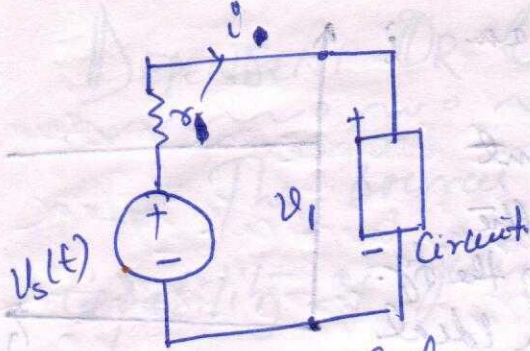
### ii) Practical Voltage Source: $\rightarrow$

A practical voltage source is the one in which the voltage across the terminals of the source keeps falling as the current through it increases. Hence, there is a finite internal resistance however small.

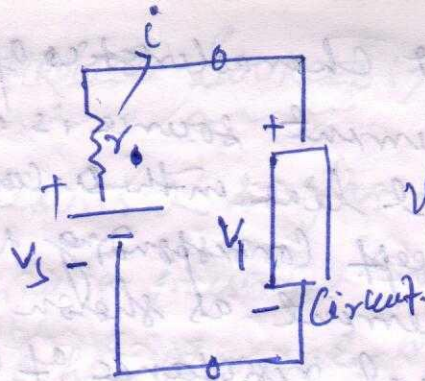
( $i-v$  characteristic of an ideal voltage source).

Thus, a practical voltage source is represented by connecting an internal resistance  $r_i$  in series with an ideal voltage source as shown in the fig. given below. Then we have the terminal voltage  $v$  at  $v = v_s - i r_i$ , where  $i$  is the current flowing through the source and  $r_i$  is the internal resistance of the source.  $i-v$  characteristics for a practical voltage source is





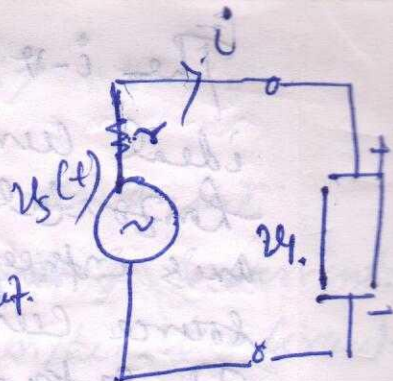
General symbol for practical voltage source,  $V_s(t)$  may be constant (DC source) or time varying.



A special case: DC voltage source.

$$V_1 < V_s$$

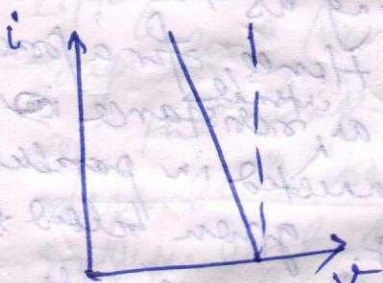
(Practical voltage sources).



A special case: Time varying source i.e. an alternating voltage source,  $V_s(t) = V_m \cos \omega t$ .

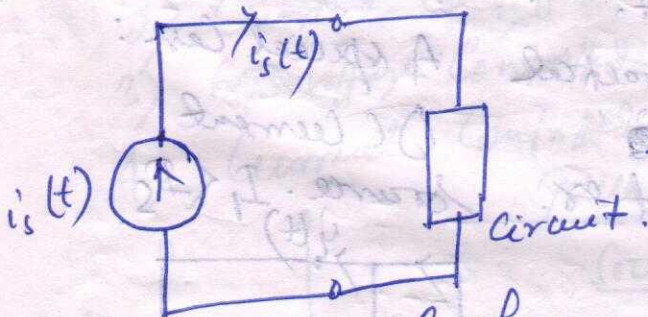
$$V_1 < V_s(t)$$

$$V_1 < V_s(t)$$

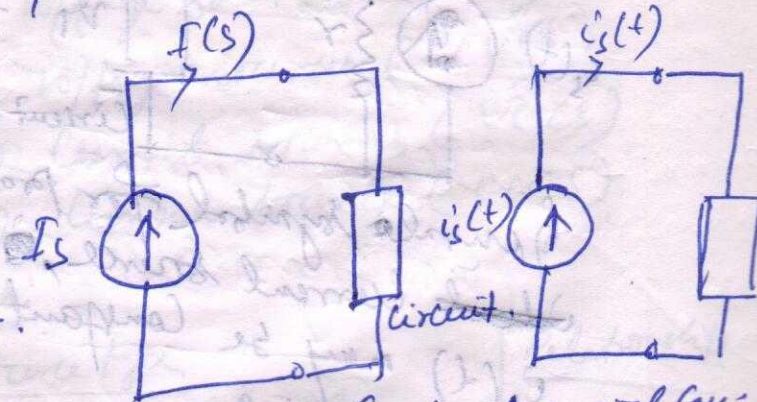


(i-v characteristic of a practical voltage source)

B. The Current source:  $\rightarrow$  An ideal current source provides a prescribed constant current regardless of the voltage across its terminals. The voltage generated by the source is determined by the circuit connected to it. The fig. given below depicts the various symbols for ideal current source.



General symbol for ideal current source,  $i_s(t)$  may be constant (DC source)

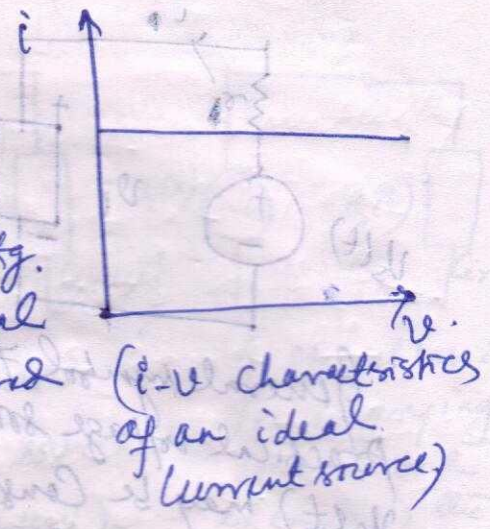


A special case: DC current source.

A special case: Time varying current source.

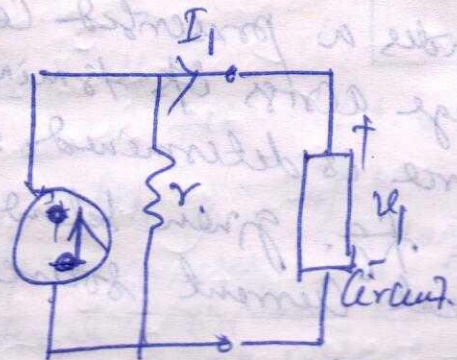
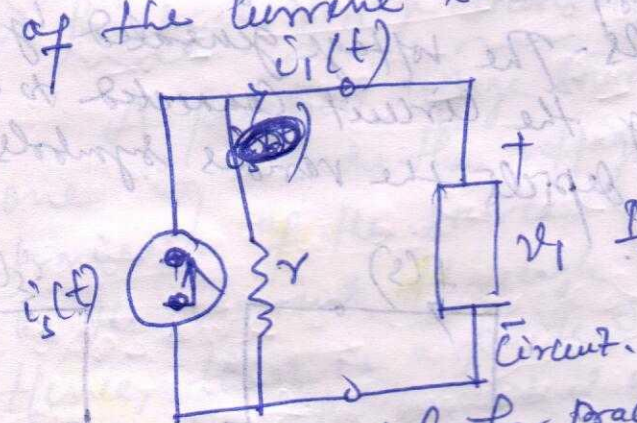


The  $i-v$  characteristics of an ideal current source is a horizontal line with a current axis intercept corresponding to the source current as shown in the fig. The internal resistance of an ideal current source is always considered infinite.



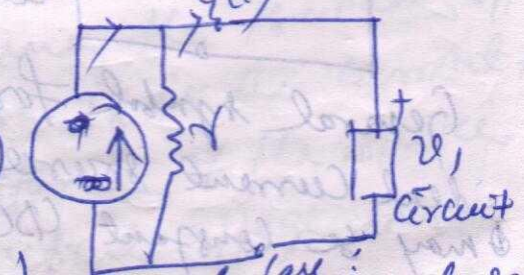
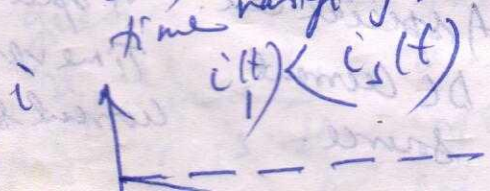
ii) Practical Current Source  $\rightarrow$

A practical current source is one in which the terminal current keeps falling as the voltage across it terminal increases. Hence, for a practical current source, there is always an internal resistance in parallel with the source as shown in the fig. given below such that the terminal current is  $i_1 = i_s - \frac{v_1}{r}$  where  $v_1$  is the terminal voltage and  $r$  is the internal resistance of the current source.



General symbol for practical current source,  $i_s(t)$  may be constant or time varying.

A special case: DC current source.  $I_1 < I_s$



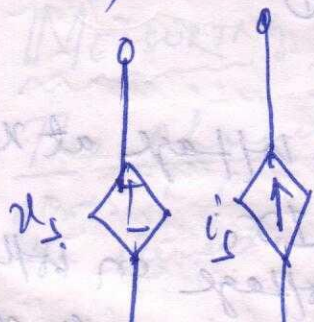


# DEPENDENT OR CONTROLLED SOURCES: →

The sources described so far have the capability of generating a prescribed voltage or current independent of any other element within the circuit. Thus, they are termed independent sources. There exists another category of sources, however, whose output (current or voltage) is a function of some other voltage or current in a circuit. These are called dependent (or controlled) sources. A different symbol, in the shape of a diamond, is used to represent dependent sources and to distinguish them from independent sources. The symbols typically used to represent dependent sources are depicted below in the fig. The dependent or controlled sources can be classified as follows and the table given below illustrates the relationship between the source voltage or current and the voltage or current it depends on -  $v_x$  or  $i_x$ , respectively - which can be any voltage or current in the circuit.

- 1) Voltage Controlled Voltage source (VCVS)
- 2) Current Controlled Voltage source (CCVS)
- 3) Voltage Controlled Current source (VCCS)
- 4) Current Controlled Current source (CCCS)

Table:

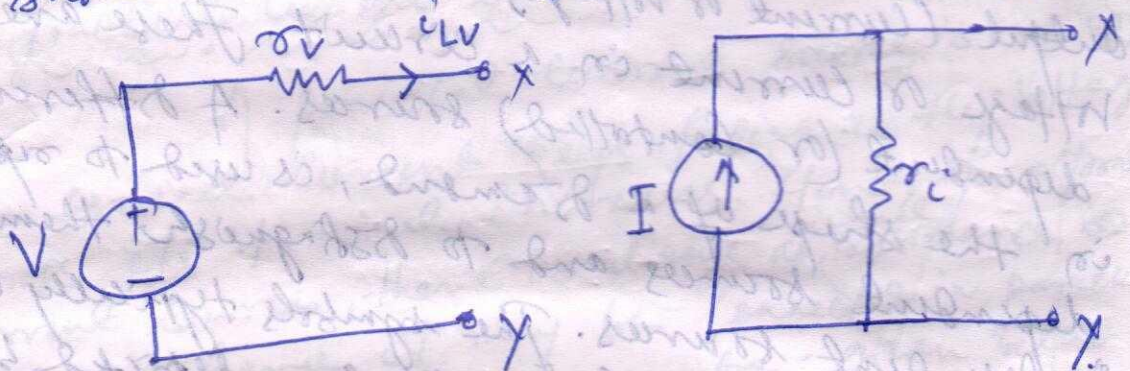


Source type	Relationship
1) Voltage Controlled Voltage source (VCVS)	$v_s = \mu v_x$
2) Current Controlled Voltage source (CCVS)	$v_s = r i_x$
3) Voltage Controlled Current source (VCCS)	$i_s = \beta i_x$
4) Current Controlled Current source (CCCS)	$i_s = \alpha i_x$



# SOURCE TRANSFORMATIONS: →

The voltage and current sources are mutually transformable. Let us consider a practical voltage source of internal resistance  $r_v$  in series with the source and a practical current source of internal resistance  $r_i$  in parallel with the source as shown on the fig. given below.



(A practical voltage source)

(A practical current source)

In the first case, the load current is given by

$$i_{LV} = \frac{V}{r_v + r_L} \quad (\text{assuming a load resistance } r_L \text{ is connected across the terminals } x-y)$$

and in the 2nd case, assuming the same load resistance  $r_L$  connected across the terminals  $x-y$ , the load current is given by:  $i_{Li} = I \cdot \frac{r_i}{r_i + r_L}$

Two sources to become identical;

$$\frac{V}{r_v + r_L} = I \cdot \frac{r_i}{r_i + r_L} \quad \text{--- (1)}$$

xy being open, the terminal voltage at x-y for the current source will be  $I r_i$  terminal voltage in both



$$V = IR_i \quad \& \quad I = \frac{V}{r_i} \quad \text{--- (2)}$$

putting this value in eqn (1), we get:

$$\frac{V}{r_i + r_L} = \frac{V}{r_i + r_L}$$

$$\therefore r_i + r_L = r_i + r_L$$

$$\text{i.e. } \boxed{r_v = r_i} \quad \text{(3)}$$

For transformation of voltage source to current source the following should be done.

Magnitude of the current source will be  $I = \frac{V}{r_i} = \frac{V}{r_v}$  as per eqn (2) & (3). and the resistance in parallel with the current source will be  $r_i = r_v$ .

The current source has  $I = \frac{V}{r_v}$  &  $r_i = r_v$ .

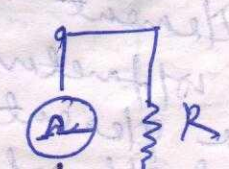
Similarly, for transformation of current source to voltage source the following should be done.

Magnitude of the voltage source would be  $V = IR_i$  as per eqn (2). and the resistance in series with the voltage source will be  $r_i = r_v$ .

The voltage source has  $V = IR_i$  &  $r_v = r_i$ .

### MEASURING DEVICES :->

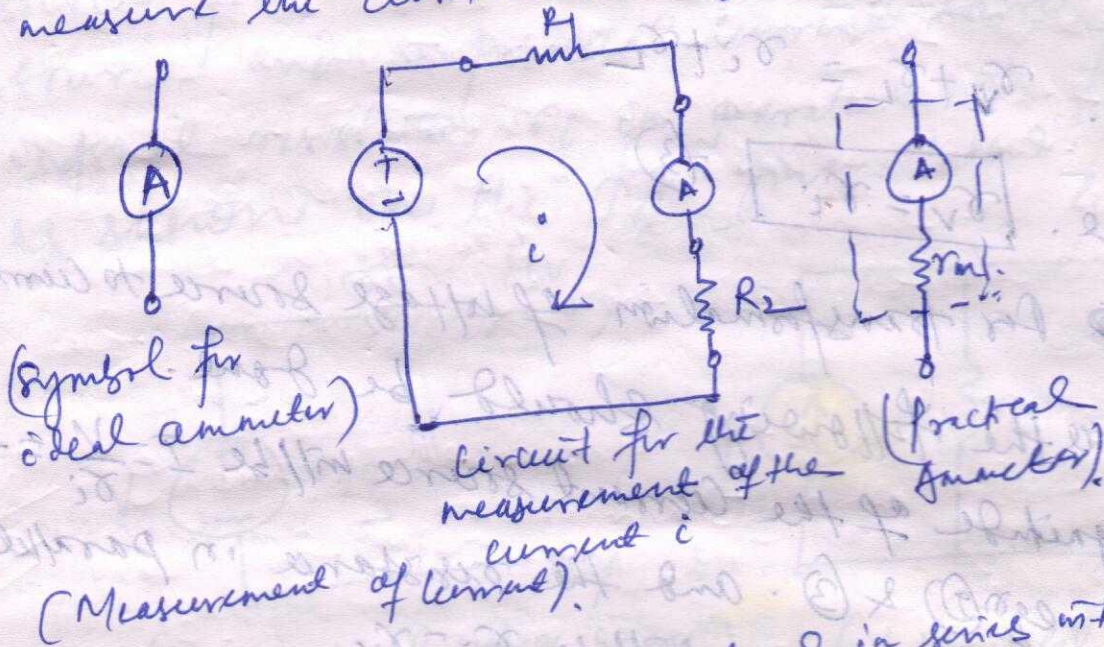
The Ohmmeter :-> The ohmmeter is a device when connected across a circuit element, can measure the resistance of the element.





Notes: → The resistance of an element can be measured only when the element is disconnected from any other circuit.

The Ammeter: → The ammeter is a device that when connected in series with a circuit element, can measure the current flowing through the element.



- Notes:
1. The ammeter must be placed in series with the element whose current is to be measured.
  2. The ammeter should not restrict the flow of current (i.e., cause a voltage drop), or else it will not be measuring the true current flowing in the circuit.
- An ideal ammeter has zero internal resistance.

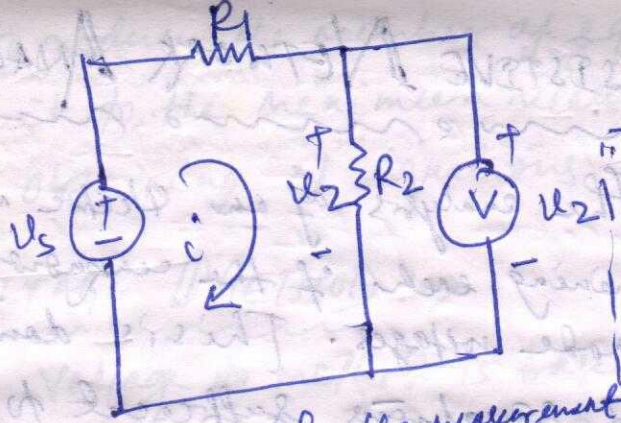
The Voltmeter: → The voltmeter is a device that can measure the voltage across a circuit element. Since voltage is the difference in potential between two points in a circuit, the voltmeter needs to be connected across the element whose voltage is to be measured. A voltmeter must also fulfill two requirements as follows:

1. The voltmeter must be placed in parallel with the element whose voltage is to be measured.
2. The voltmeter should draw no current away from the element whose voltage it is measuring, or else it will not be measuring the true voltage across the element.

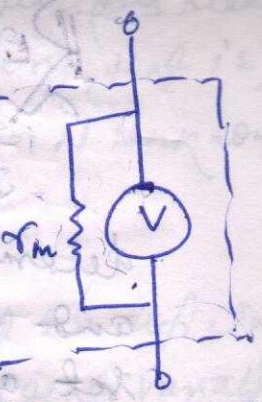




(Ideal voltmeter)



(Circuit for the measurement of the voltage  $V_2$ )

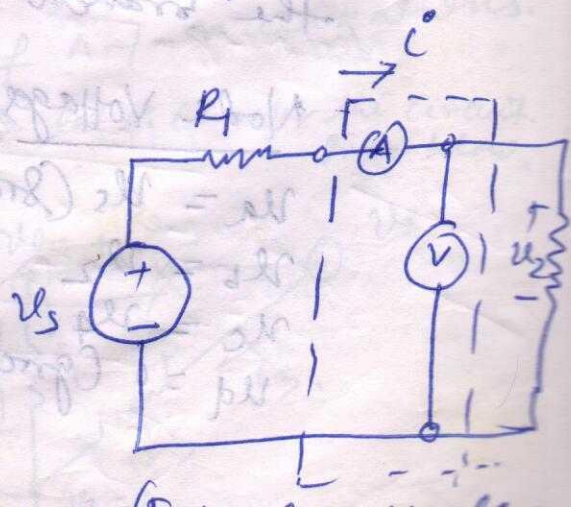
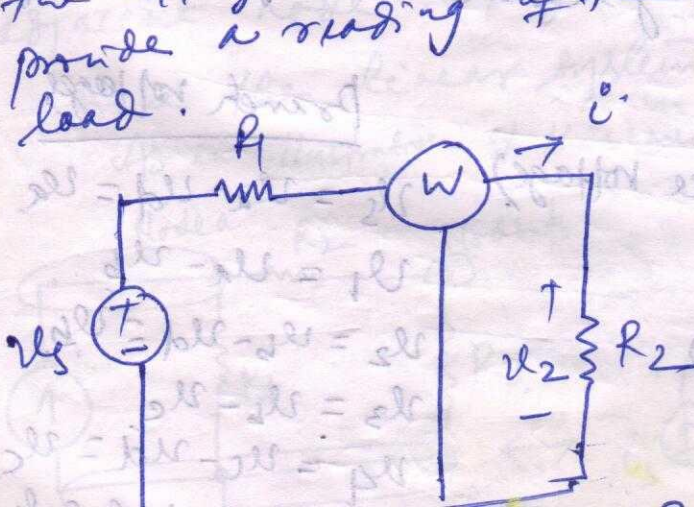


(Practical voltmeter)

(Measurement of voltage)

The practical restrictions apply applied to the measuring devices do not necessarily pose a limit to the accuracy of the measurements obtainable with practical measuring devices, as long as the internal resistance of the measuring devices is known.

The wattmeter: All the considerations that pertain to practical ammeters and voltmeters can be applied to the operation of a wattmeter, an instrument that provides a measurement of the power dissipated by a circuit element, since the wattmeter is in effect made up of a combination of a voltmeter & ~~wattmeter~~ an ammeter. In effect, the wattmeter measures the current flowing through the load and, simultaneously, the voltage across it and multiplies the two to provide a reading of the power dissipated by the load.





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A.K. Theraja.  
S. Chand.